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3E1416

B. Tech. (Sem. III) (Main & Back) Examination, January - 2013
3AI6 Advanced Engg. Mathematics
(Common for Mech., AE & PI)

Time : 3 Hours] [Total Marks : 80
[Min. Passing Marks : 24

Attempt any five questions, selecting one question from each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.
Units of quantities used/ calculated must be stated clearly.

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. Nil 2. NIL

UNIT - I

1 (a) Find the Fourier series for $f(x)=x+x^2, -\pi<x<\pi$.

Hence show that $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ 8

(b) Obtain the expansion for y from the following table upto the first harmonic :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

8

OR

1 (a) Find the Fourier sine and cosine transform of $f(x)$,

where $f(x)=1$ *for* $0<x<a$
$=0$ *for* $x>a$

8

(b) Solve $\frac{du}{dt}=\frac{d^2u}{dx^2}$, given that $u_x(0,t)=0$ and

$$u(x,0)=\begin{cases}x, 0\le x\le 1\\ 0, x>1\end{cases}$$

$u(x,t)$ is bounded and $x>0,\ t>0$. 8

UNIT - II

2 (a) An inulated rod of length l has i ends A and B kept at 0^oC and 100^oC respectively until steady state conditions prevail. If the temperature of B is then suddenly reduced to 0^oC and kept so, while that of end A is maintained, find the temperature $u(x,t)$ at distance x from A at time t. 8

(b) Find the laplace transform of $\frac{1}{t}\sin at$. 8

OR

2 (a) Find the Inverse Laplace transform of $\frac{11s^2-2s+5}{(s-2)(2s-1)(s+1)}$. 8

(b) Solve the following equation :

$(D^2+3D+2)x(t)=1,\ x(0)=0,\ D(x)=0$ at $t=0$. 8

UNIT - III

3 (a) If $f(z)=\begin{cases}\frac{x^3y(y-ix)}{x^6+y^2}, & z\neq 0\\ 0, & z=0\end{cases}$

prove that $\frac{f(z)-f(0)}{z-0}\to 0$ as $z\to 0$ along any radius vector but not as $z\to 0$ along the curve $y=ax^3$. Is this function differentiable at $z=0$? 8

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. 8

OR

3 (a) Evaluate the integral $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\, dz$, where c is the circle $|z| = 3$, by using Cauchy's integral formula. 8

(b) Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane. 8

UNIT - IV

4 (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by

$y = y_0 \sin^3 \pi x/l$.

It is released from rest from this position. Find the displacement $y(x,t)$. 8

(b) Prove $\left(1 - 2xt + t^2\right)^{-1/2} = \sum_{n=0}^{\infty} P_n(x) t^n$.

8

OR

4 (a) Prove that $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x), n \geq 0$.

8

(b) Prove that $\frac{d}{dx}\left[J_n^2(x) + J_{n-1}^2(x)\right] = 2\left[\frac{n}{x} J_n^2(x) - \frac{n+1}{x} J_{n+1}^2(x)\right]$

8

UNIT - V

5 (a) The ordinates of the normal curve are given by the following table :

x	0.0	0.2	0.4	0.6	0.8
y	0.3989	0.3910	0.3683	0.3332	0.2897

Evaluate :

(i) $y(0.25)$

(ii) $y(0.62)$

(iii) $y(0.43)$ 8

(b) Use Lagrange's interpolation formula to find y when $x = 2$, given that

$x =$	0	1	3	4
$y =$	5	6	50	105

8

OR

5 (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using

(i) Trapezoidal rule

(ii) Simpson's $\frac{1}{3}$ rule

(iii) Simpson's $\frac{3}{8}$ rule. 8

(b) Define the operators δ and μ and prove that

$$\delta[f(x)g(x)] = \mu[f(x)]\delta[g(x)] + \mu[g(x)]\delta[f(x)]$$

8