3E1416

Roll No. : ______________________

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B.Tech. (Sem. III) (Main/Back) Examination, January - 2012
Automobile Engg.
3AE6 Advanced Engineering Mathematics
(Common for ME/PI)

Time : **3** Hours]

[Total Marks : **80**

[Min. Passing Marks : **24**

Instructions to Candidates :

Attempt any ***five questions*** *selecting* ***one question*** *from* ***each unit. All*** *questions carry* ***equal*** *marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.*

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. Nil

2. Nil

UNIT - I

1 (a) Find the Fourier series of

$$f(x) = x^2 \quad , -\pi < x < 0$$
$$= -x^2 \quad , 0 < x < \pi$$

In the interval given. 6

(b) Obtain the Fourier cosine series for

$$f(x) = x^2, \quad 0 \le x < 1$$
$$= 1, \quad 1 \le x < 2$$

5

(c) Find the Fourier transform of

$$f(t) = \begin{cases} -(1+t), & -1 \le t < 0 \\ t-1, & 0 < t \le 1 \\ 0, & |t| > 1 \end{cases}$$

5

OR

1 (a) State and prove the convolution theorem for Fourier transforms. 2, 6

(b) Use Fourier transforms to solve :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, \ t > 0 \text{ and}$$

$$u(x, 0) = e^{-2|x|}, -\infty < x < \infty$$

8

UNIT - II

2 (a) Solve the steady state temperature distribution equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \ 0 < x < a, \ 0 < y < b$$

subject to $u(x, 0) = f(x), u(x, b) = g(x), 0 < x < a$ and $u(0, y) = 0, u(a, y) = 0, 0 < y < b$.
Use separation of variables method.

8

(b) (i) Find the Laplace transform of $\sin(3t+2)$

4

(ii) Find the Inverse Laplace transform of $\frac{1}{s(s^2+9)}$.

4

OR

2 (a) Use Laplace transforms solve $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = e^{3t}$,

$y(0) = 0$ and $y'(0) = 2$.

8

(b) (i) Find the Laplace transform of $\frac{1}{t}(1-\cos bt)$.

4

(ii) Obtain the Inverse Laplace transform of $\frac{1}{(s^2+9)^2}$.

4

UNIT - III

3 (a) Determine the analytic function $f(z) = u + iv$ where $u + v = e^x(\cos y + \sin y)$.

8

(b) Show that $u = \left(y^3 - 3x^2 y\right)$ is harmonic. Find the corresponding conjugate function $v(x, y)$ so that $f(z) = u + iv$ is analytic. **4**

(c) Under the mapping $f(z) = z^2$, find the image of the region bounded by the lines $x = 1$, $y = 1$ and $x + y = 1$. **4**

OR

3 (a) State and prove Cauchy's integral theorem. **1, 4**

(b) Find the poles of the function $f(z) = \dfrac{e^z}{(z - \sin z)}$

Also, find the principal part in the Laurent expansion of $f(z)$ about $z = 0$. **2, 3**

(c) Use contour integration to evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{(3 + 2\sin\theta)}$ **6**

UNIT - IV

4 (a) A string is stretched between the fixed points $(0, 0)$ and $(l, 0)$ and released from rest from the initial deflection given by

$$f(x) = \begin{cases} \dfrac{2k}{l} x, & 0 < x < \dfrac{l}{2} \\ \dfrac{2k}{l}(l - x), & \dfrac{l}{2} < x < l \end{cases}$$

Find the deflection of the string at any time t. **8**

(b) Obtain the steady state temperature distribution in a semi-circular plate whose bounding diameter is kept at 0° C while the circumference is kept at 50° C. **8**

OR

4 (a) Solve in series : $\left(1 - x^2\right)\dfrac{d^2 y}{dx^2} - 2x\dfrac{dy}{dx} + 6y = 0$ **8**

(b) Express $f(x) = 4x^3 + 3x^2 + 2x - 6$ in terms of Legendre polynomials. **4**

(c) Evaluate : $\int_0^a x\,J_0(rx)\,dx$ 4

UNIT - V

5 (a) Given

x :	2.5	3.0	3.5	4.0	4.5	5.0
y :	24.145	22.043	20.225	18.644	17.262	16.047

Obtain y for (i) $x=2.6$ (ii) $x=3.7$ and (iii) $x=4.8$. State the formulae used.

3×3=9

(b) Given the following data :

x	300	304	305	307
y	2.4771	2.4829	2.4843	2.4871

Use Lagrange interpolation formula to obtain y for $x=306$. 7

OR

5 (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ from the following data :

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.0000	1.0247	1.0488	1.0723	1.0954	1.1180	1.1401

for $x=1.00$.

3, 3

(b) Evaluate numerically $\int_0^{\pi/2} \sqrt{(\cos\theta)}\,d\theta$.

6

(c) Show that $\mu\,\delta = \frac{1}{2}(\Delta+\nabla)$, with usual meanings for symbols used.

4
