3E1417

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B.Tech. (Sem. III) (Main) Examination, February/March - 2010
Mechanical Engineering
(3ME 6 Advanced Engineering Mathematics)

Time : 3 Hours] [Maximum Marks : 80
[Min. Passing Marks : 24

Attempt overall five questions selecting one question from each Unit. All questions carry equal marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used / calculated must be stated clearly.)

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. Nil 2. Nil

UNIT - I

1 (a) Find the Fourier series for the function defined as :

$$f(x) = \begin{cases} -1, & for\ -\pi \le x < 0 \\ 0, & for\ x = 0 \\ 1, & for\ 0 < x < \pi \end{cases}$$

Hence prove that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$

(b) Obtain the constant term and the coefficients of first sine and cosine terms in the Fourier expansion of y as given in the following table :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

OR

1 (a) A string is stretched between the fixed points (0, 0) and (l, 0) and released from the position $y = A\sin\frac{\pi x}{l}$ at $t = 0$ find $y(x, 5)$.

(b) Find the Fourier series of the function $f(x)$, $-l < x < l$ defined as below :

$$f(x) = \begin{cases} \frac{l}{4}, & when \ -l < x < -\frac{l}{2} \\ \frac{x^2}{l}, & when \ -\frac{l}{2} < x < \frac{l}{2} \\ \frac{l}{4}, & when \ \frac{l}{2} < x < l \end{cases}$$

UNIT - II

2 (a) Find the Laplace transform of $\sin\sqrt{t}$. Hence show that

$$L\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right) = \left(\frac{\pi}{s}\right)^{\frac{1}{2}} e^{-\frac{1}{4s}}.$$

(b) Use Laplace transform technique to solve

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = t$. Given that $y(0) = -3$, $y(1) = -1$.

OR

2 (a) Prove that $L\left(\frac{\sin t}{t}\right) = \tan^{-1}\left(\frac{1}{s}\right)$ and hence find

$L\left(\frac{\sin at}{t}\right)$, does $L\left(\frac{\cos at}{t}\right)$ exist ? Also prove that

$$\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}.$$

(b) Apply convolution theorem to find inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$.

UNIT - III

3 (a) Prove the following Recurrence relations for Bessel function J_n :

(i) $x J_n'(x) = n J_n(x) - x J_{n+1}(x)$

(ii) $x J_n'(x) = -n J_n(x) + x J_{n-1}(x)$.

(b) State and prove Rodrigue's formula.

OR

3 (a) Prove that

(i) $\frac{d}{dx}\left[J_n^2 + J_{n+1}^2\right] = 2\left[\frac{n}{x} J_n^2 - \frac{n+1}{x} J_{n+1}^2\right]$

(ii) $J_0^2 + 2\left[J_1^2 + J_2^2 + J_3^2 + ...\right] = 1$

(b) Prove that

(i) $(n+1) P_{n+1} = (2n+1)\, x\, P_n - nP_{n-1}$

(ii) $(2n+1) P_n = P''_{n+1} - P'_{n-1}$.

UNIT - IV

4 (a) Given that

θ =	0°	5°	10°	15°	20°	25°	30°
tan θ =	0.0000	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

Find $\tan 3°$, $\tan 16°$ and $\tan 28°$ by using appropriate interpolation formula.

(b) Evaluate : $\Delta^6 (ax-1)(bx^2-1)(cx^3-1)$.

OR

4 (a) Using Lagrange's interpolation formula find the value of y when $x = 10$ from the following data :

x	5	6	9	11
y	12	13	14	16

(b) Find the real root of the equation $x^2 + y \sin x = 0$ correct to four places of decimals by using Newton-Raphson's method.

UNIT - V

5 (a) Use Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule to evaluate the following $\int_0^1 \frac{dx}{1+x^2}$. Hence obtain the approximate value of π in each case.

(b) Solve the following system by Gauss-Seidal iteration method :

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

OR

5 (a) Find the first derivates at $x = 1.2,\ 1.8$ and 1.4 from the following table :

x	1	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.1280	0.5440	1.2960	2.4320	4.00

(b) Using Runge-Kutta method, find the approximate value of

$$y(0.2) \text{ if } \frac{dy}{dx} = x + y^2$$

given that $y = 1$ when $x = 0$ taking $h = 0.1$