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B.Tech. (Sem. III) (Main/Back) Examination, February - 2010
(Common for Mech., P. & I. & Automobile Engg.)
(3AE6 Advance Engineering Mathematics)

Time : **3** Hours] [Total Marks : **80**
[Min. Passing Marks : **24**

*Attempt overall **five** questions in all. Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitable be assumed and stated clearly.*

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. Nil 2. Nil

1 (a) Find a Fourier series for the function $f(x) = e^{-ax}$ in the interval $-\pi < x < \pi$, where 'a' is a constant and hence deduce a series for $\frac{\pi}{\sin h\, \pi}$. 8

(b) Find the Laplace transform of $\sin\sqrt{x}$. Hence show that $L\left[\frac{\cos\sqrt{x}}{\sqrt{x}}\right] = \left[\frac{\pi}{s}\right]^{1/2} e^{-1/4s}$. 8

OR

1 (a) Find half range cosine series for the function $f(x) = (x-1)^2$; $0 < x < 1$. Hence show that $\frac{\pi^2}{8} = \left(\frac{1}{1^2} + \frac{1}{3^2} + ...\right)$. 8

(b) Find the Fourier transforms of $f(x)=\begin{cases}1, & |x|\le a\\ 0, & |x|>a\end{cases}$. Also evaluate $\int_{-\infty}^{\infty}\frac{\sin\lambda a\cos\lambda x}{\lambda}d\lambda$. Deduce the value of $\int_{0}^{\infty}\frac{\sin u}{u}du$. 8

2 (a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $\lambda x(l-x)$, find the displacement of the string at any distance x from one end at any time t. 8

(b) Solve by Laplace transform method,

$(D^2+9)y=\cos 2t,\ y(0)=1,\ y(\pi/2)=-1$, where $D=\frac{d}{dt}$. 8

OR

2 (a) Find the solution of the differential equation $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ subjected to the conditions : (i) u is not infinite for $t\to\infty$, (ii) $\frac{\partial u}{\partial x}=0$ for $x=0$ and $x=l$, (iii) $u=lx-x^2$ for $t=0$ between $x=0$ and $x=l$. 8

(b) Find the bounded solution $y(x,t)$, $0<x<1$, $t>0$ of the boundary value problem : $\frac{\partial y}{\partial x}-\frac{\partial y}{\partial t}=1-e^{-t}$, $y(x,0)=x$ using Laplace transform method. 8

3 (a) Show that the function $u = e^x (x \cos y - y \sin y)$ is harmonic and determine its conjugate function.

4+4

(b) Evaluate (i) $\oint_C \frac{z^2}{(z-2)(z-1)^2} dz$, where C is the circle $|z| = \frac{5}{2}$. (ii) $\oint_C z^2 e^{1/z} dz$, where C is the circle $|z| = 1$.

4+4

OR

3 (a) Show that under the transformation $w = \frac{z-i}{z+i}$, real axis in the z-plane is mapped into the circle $|w| = 1$. What portion of the z-plane corresponds to the interior of the circle ?

8

(b) Expand the function $\frac{z}{(z^2+1)(z^2+4)}$ for

(i) $|z| < 1$ (ii) $1 < |z| < 2$ in Laurent's series.

8

4 (a) Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in xy plane, $0 < x < a$ and $0 < y < b$ satisfying the following boundary conditions

$$u(x, 0) = 0,\ u(x, b) = 0,\ u(0, y) = 0,\ u(a, y) = f(y).$$

8

(b) Show that

(i) $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$

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(ii) $2nJ_n(x) = x\left[J_{n-1}(x) + J_{n+1}(x)\right]$

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OR

4 (a) Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials. 8

(b) Prove that $\int x\, J_0^2(x)dx = \frac{x^2}{2}\left[J_0^2(x) + J_1^2(x)\right] + C$. 8

5 (a) Prove that $\delta[f(x)\,g(x)] = \mu[f(x)]\,\delta[g(x)] + \mu[g(x)]\,\delta[f(x)]$. 8

(b) Evaluate $\int_4^{5.2} \log_e x\, dx$ by Simpson's $\frac{1}{3}$ rule. After finding the true value of integral compare the error. 8

OR

5 (a) If $y = f(x)$ and $y_n = f(x_0 + nh)$, prove that, if powers of h above h^6 be neglected,

$$f'(x_0) = \frac{3}{4h}\left[(y_1 - y_{-1}) - \frac{1}{5}(y_2 - y_{-2}) + \frac{1}{45}(y_3 - y_{-3})\right]$$

8

(b) Use Lagrange's interpolation formula to find y when $x = 2$, given that

$x =$	0	1	3	4
$y =$	5	6	50	105

8