

OR

3. a) Compute the value of $\int_0^6 \frac{dx}{1+x^2}$ by (8)

i) Trapezoidal rule

ii) Simpson's $\frac{1}{3}$ rule

b) Use picard method to solve

$$\frac{dy}{dx} = x+y^2, x_0 = 0 \text{ when } y_0 = 0. \quad (8)$$

Unit - IV

4. a) If α and β be two different roots of $J_n(x) = 0$, (8)

prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$

$$\text{and } \int_0^1 x J_n(\alpha x) J_n(\beta x) dx = \frac{1}{2} \{J_n'(\alpha)^2\}$$

b) Prove by using generating form of Legendre's function

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3). \quad (8)$$

OR

4. a) Prove that : (8)

$$i) J_{\frac{1}{2}x} = \sqrt{\frac{2}{\pi x}} \sin x$$

$$ii) J_{\frac{1}{2}x} = \sqrt{\frac{2}{\pi x}} \cos x$$

b) Prove that : $\int_{-1}^1 x^3 P_4(x) dx$

where $P_4(x)$ is Legendre polynomial of degree four. (8)