3E1486

Roll No. : ______________________

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B. Tech. (Sem. III) (Main/Back) Examination, January - 2012
Electrical Engg. (Common with Electrical & Electronics)
3EE6 Mathematics (Common with 3EE6.1 & 3EX1)

Time : **3** Hours]

[Maximum Marks : **80**

[Min. Passing Marks : **24**

Instructions to Candidates :

Attempt any ***five questions*** *selecting* ***one question*** *from* ***each unit****. All questions carry* ***equal*** *marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.*

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. NIL

2. NIL

UNIT - I

1 (a) Find the Laplace transform of the following functions :

(i) $3+2t-4t^3+\cos 4t-3e^{-2t}$

(ii) $\frac{1}{t}(1-\cos t)$.

8+8=16

(b) Find the inverse Laplace transform of the following functions :

(i) $\frac{1}{s^2}-\frac{2}{s+3}-\frac{1}{s^2+4}$

(ii) $\frac{s}{s^4+4a^4}$.

8+8=16

OR

1 (a) Using Laplace transform technique to solve the following differential equation :

$$\left(D^2+9\right)y=\cos 2t,\ D\equiv\frac{d}{dt}$$

with $y(0)=1,\ y\left(\frac{\pi}{2}\right)=-1$

8

(b) Solve the following partial differential equation by using Laplace transform technique

$$\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}, u=u(x,t)$$

with $u(0,t)=0=u(5,t)$ and $u(x,0)=10\sin 4\pi x$.

8+8

UNIT - II

2 (a) Find the inverse Fourier sin transform of $\left\{\frac{e^{-as}}{s}\right\}$ and hence deduce $F_s^{-1}\left\{\frac{1}{s}\right\}$.

8

(b) Solve the integral equation :

$$\int_0^\infty f(x)\cos sx\,dx=\begin{cases}1-s, & \text{when } 0\le s\le 1\\ 0, & \text{when } s>1\end{cases}$$

8

OR

2 (a) Find the discrete Fourier transform of the sequence $\{1, 2, 3, 4\}$.

(b) Solve the following partial differential equation by using Fourier transform technique :

$$\frac{\partial V}{\partial t}=\frac{\partial^2 V}{\partial x^2}$$

if

(i) $V_x(0,t)=0$

(ii) $V(x,0)=\begin{cases}x, & 0\le x\le 1\\ 0, & x>1\end{cases}$ and

(iii) $V(x,t)$ is bounded $x>0, t>0$

UNIT - III

3 (a) Find the Fourier series of the function

$f(x)=x+x^2$

in the interval $(-\pi,\pi)$ and show that

$$\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+.....$$

6+2=8

(b) Obtain the cosine series of $\sin x$ in the interval $0 < x < \pi$

8

OR

3 (a) Find the external for the functional

$$\int_1^2 \frac{\sqrt{1+y'^2}}{x}\,dx \quad y(1)=0, y(2)=1$$

8

(b) Find a function $y(x)$ for which $\int_0^1 \left(x^2 - y'^2\right)dx$ is stationary, given that $\int_0^1 y^2 dx = 2, y(0)=0, y(1)=0$

8

UNIT - IV

4 (a) State and prove Cauchy-Riemann equations.

8

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ respectively into the points $w = i, 0, -i$. For this transformation, find the image of $|z| \le 1$.

5+3=8

OR

4 (a) Evaluate the integral

$$\int_0^{1+i} z^2 dz$$

8

(b) Evaluate : $\int_C \frac{(1-2z)}{z(z-1)(z-2)}\,dz$ where C is the circle $|z| = 2.5$.

8

UNIT - V

5 (a) Expand $\frac{z^2-4}{(z+1)(z+4)}$ in Laurent's series for the regions

(i) $|z| < 1$ (ii) $1 < |z| < 4$ (iii) $|z| > 4$

8

(b) Find the poles of the following functions, find also the order of each poles :

(i) $\frac{1}{1+z^4}$ (ii) $\frac{1}{\sin z - \cos z}$.

4+4=8

OR

5 (a) Find the residue of $\frac{z^2 - 2z}{(z+1)^2 \left(z^2+4\right)}$ at all its poles in the finite plane.

8

(b) Prove that :

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} \, dx = \frac{5\pi}{12}$$

8