4E 2989

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B.Tech. IV Semester (Main/Back) Examination - 2012

Electronics & Comm.

4EC5 Random Variables & Stochastic Processes

Time : 3 Hours **Maximum Marks : 80**

Min. Passing Marks : 24

Instructions to Candidates:

*Attempt any **Five questions** selecting **one question** from **each unit**. All questions carry **equal marks**. (Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly.) Units of quantities used/calculated must be stated clearly.*

Unit - I

1. a) A_1 and A_2 take turns in throwing of two dice, the first to throw I will be awarded prize. If A has first turn. Show their chances of winning one in the ratio 9:8. **(8)**

b) State and prove Baye's Theorem. **(8)**

OR

a) In a communication system *a* '0' or '1' is transmitted correctly with probabilities p_1 and p_2 respectively. Due to the noise in the channel, *a* '0' can be received as one or one can be received as '0' with probability P_3. If *a* '1' is received what is the probability that '*a*' '1' was transmitted. **(6)**

b) The chance that a doctor will diagnose a disease correctly is 70%. The chances of death of a patient after correct is 35%. While after wrong diagnosis it is 80%. If patient dies after taking his treatment find the probability that he was diagnosed

i) Wrongly

ii) Correctly. **(10)**

Unit - II

2. a) The probability that a person will die in the time interval (t_1, t_2) is given by

$$P[t_1 \le t \le t_2] = \int_{t_1}^{t_2} f(t)\, dt .$$

Where $f(t) = \begin{cases} 3\times10^{-9} t^2 (100-t)^2 \; ; & 0 \le t \le 100 \\ 0 \; ; & \text{elsewhere} \end{cases}$

Determine

i) The probability that a person will die between the ages 60 and 70.

ii) The probability that he will die between those ages assuming he lived up to 60. **(8)**

b) Prove for Binomial Distribution

$$\left[\frac{d\mu_r}{dp} + nr\mu_{r-1}\right] pq = \mu_{r+1}$$

also find first four central moments. **(8)**

OR

a) Prove that Poisson Distribution is a limiting case of Binomial Distribution. **(10)**

b) If the skulls are classified as A,B,C according as the index is 75, between 75 and 80. Over 80 find approximately (assuming Normal Distribution) the mean and SD of the series in which A are 58%, B are 38%, and C are 4% being given and **(6)**

$$f(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-x^2/2}\, dx \text{ then } f(0.20) = .08, \quad f(1.75) = .46$$

Unit - III

3. a) The joint probability density function of a bivariate random variable (x, y) is given by

$$f_{xy}(x,y) = \begin{cases} \lambda(x+y) \; ; & 0 < x < 3,\; 0 < y < 3 \\ 0 \; ; & \text{otherwise} \end{cases} \qquad \lambda \text{ is a constant}$$

i) Find the value of λ

ii) Find the marginal probability density function of X and Y.

iii) Are X and Y independent. **(8)**

b) If X and Y are two independent exponential random variables with parameter 1. Let U = X+Y and V = X-Y. Then find the joint and marginal probability density function of U and V. **(8)**

OR

a) The useful life of a certain kind of tube is a random variable having the probability density

$$f_X(x) = \begin{cases} \frac{20000}{(x+1000)^3}, & x > 0 \\ 0 & \text{otherwise} \end{cases} \quad \textbf{(8)}$$

If two tubes operate Independently. Then find

i) The joint probability density of X and Y.

ii) P (X < 100, Y < 100)

b) Let X and Y are two independent random variables with probability density function $f_{xy}(x,y)$ then find pdf of (i) $z = X+Y$ (ii) $z = XY$ (iii) $z = \frac{X}{Y}$ **(8)**

Unit - IV

4. a) Find M.g.f. of Normal Distribution and hence find the mean and variance of Normal Distribution. **(8)**

b) Find coefficient of correlation for the following data

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

(8)

OR

a) A random variable X having a following p.d.f. $f(x) = \frac{1}{2\alpha} e^{-\frac{|x-\alpha|}{\alpha}} \quad -\infty < x < \infty$

Find **(8)**

(i) M.g.f. of X

(ii) E(X)

(iii) V(X)

b) State and prove chebyshev's inequality. **(8)**

Unit - V

5. a) $X(t) = A \cos (w_0 t + \phi)$ is a random process where A and w_0 is a constant and ϕ is a uniform random variable over $(-\pi, \pi)$. Show that X(t) is ergodic in both the mean and auto correlation function. **(8)**

b) Let {X(t)} and {Y(t)} are two random processes given by

$X(t) = U \cos (W_0 t + \theta)$ and $Y(t) = U \sin (W_0 t + \theta)$. Where U and W_0 are constants and θ is a random variable over $(0, 2\pi)$

(i) Find the cross correlation function

(ii) Verify $R_{XY}(-\tau) = R_{YX}(\tau)$. **(8)**

OR

a) If X(t) is a random process with constant mean μ and if $\overline{X} = \frac{1}{2T}\int_{-T}^{T} X(t)\,dt$ then {X(t)} is mean ergodic provided $\mathrm{Var}\,\left|\overline{X}\right| = 0$ as $T \to \infty$. **(8)**

b) The auto correlation function of a random process is $R_X(\tau) = e^{-\tau^2/2\sigma^2}$. .Find the power spectral density and the normalized average power content. **(8)**
