4E 2137

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**4E 2137**

# B.Tech. IV Semester (Main/Back) Examination - 2012

## Electronics & Comm.

## 4EC1 Mathematics – IV

## Common 4EC1, 4EI6.3, 4AI1, 4BM6.3, 4CRE5

**Time : 3 Hours** **Maximum Marks : 80**

**Min. Passing Marks : 24**

***Instructions to Candidates:***

*Attempt any **Five questions** selecting **one question** from **each unit**. All questions carry **equal marks**. (Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/ calculated must be stated clearly.)*

### Unit - I

1. a) Evaluate $\Delta^6 (ax\text{-}1)(bx^2\text{-}1)(cx^3\text{-}1)$. **(5)**

   b) Use stirting formula to find $y_{28}$ given $y_{20} = 49225$ $y_{25} = 48316$ $y_{30} = 47236$ $y_{35} = 45926$ $y_{40} = 44306$ **(5)**

   c) Find the value of $f(5)$ from the following table by using Lagrange's interpolation formula. **(6)**

| $x$: | 1 | 2 | 3 | 4 | 7 |
|---|---|---|---|---|---|
| $f(x)$: | 2 | 4 | 8 | 16 | 128 |

**OR**

1. a) Given the following data **(8)**

| $x$ | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|
| $10^5 u_x$ | 23967 | 28060 | 31788 | 35209 | 38368 |

   Evaluate $u_{10.5}$, $u_{12.5}$ and $u_{13.5}$ by applying suitable interpolation formula stating the formula used.

   b) i) Find the missing term from the following table **(4)**

| x : | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| f(x) : | 2 | 5 | 7 | ? | 32 |

ii) Find the form of the function given by the following table: **(4)**

| x : | 3 | 2 | 1 | -1 |
|---|---|---|---|---|
| y : | 3 | 12 | 15 | -21 |

**Unit - II**

2. a) Use simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule to avaluate the following : **(8)**

$$\int_0^1 \frac{dx}{1+x^2}$$

Hence obtain the approximate value of $\pi$ in each case.

b) Find $f'(0.02), f'(0.05)$ from the following table: **(8)**

| x : | .01 | .02 | .03 | .04 | .05 | .06 |
|---|---|---|---|---|---|---|
| f(x) : | .1023 | .1047 | .1071 | .1096 | .1122 | .1148 |

**OR**

2. a) Using Runge-kutta method find the approximate value of y(0.2) if $\frac{dy}{dx} = x + y^2$ given that y=1 when x=0, h=0.1. **(8)**

b) Use Milne's predictor - corrector method to solve the following equation. **(8)**

$\frac{dy}{dx} = x + y$ with y(0) = 0, h = 0.1

Compute the value of y for $0.4 \leq x \leq 0.6$.

**Unit - III**

3. a) Prove that **(8)**

i) $x\, J'_n(x) = n\, J_n(x) - x\, J_{n+1}(x)$

ii) $x\, J'_n(x) = x\, J_{n-1}(x) - n\, J_n(x)$

iii) $zn\, J_n(x) = x\,[J_{n-1}(x) + J_{n+1}(x)]$

b) Prove that **(8)**

$$P_n\left(-\frac{1}{2}\right) = P_0\left(-\frac{1}{2}\right) P_{2n}\left(\frac{1}{2}\right) + P_1\left(-\frac{1}{2}\right) P_{2n-1}\left[\frac{1}{2}\right] + \ldots\ldots\ldots + P_{2n}\left(-\frac{1}{2}\right) P_0\left(\frac{1}{2}\right)$$

**OR**

**3.** a) i) Prove that **(8)**

$$\frac{d}{dx}\left[J_n^2 + J_{n+1}^2\right] = 2\left[\frac{n}{x}J_n^2 - \frac{n+1}{x}J_{n+1}^2\right]$$

ii) $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \ldots) = 1$

b) Prove that **(8)**

$$P_{n+1}' + P_n^1 = P_0 + 3P_1 + 5P_2 + \ldots\ldots + (2n+1)P_n$$

## Unit - IV

**4.** a) Ten competitors in a beauty contest got marks by three judges in the following order. **(5)**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| First Judge : | 1 | 6 | 5 | 10 | 3 | 2 | 4 | 9 | 7 | 8 |
| Second Judge : | 6 | 4 | 9 | 8 | 1 | 2 | 3 | 10 | 5 | 7 |
| Third Judge : | 3 | 5 | 8 | 4 | 7 | 10 | 2 | 1 | 6 | 9 |

Use the rank correlation coefficient to discuss which pair of judges have the nearest approach to common testes in beauty.

b) A factory produces razor blades. The probability of its being defective in $\frac{1}{500}$. In 10,000 packets of 10 blades each. Calculate the approximate number of packet **(5)**

a) Having no defective b) one defective blade c) two defective blade
(Given $e^{-8.02} = 0.9802$)

c) A perfect cubic die in thrown a large number of times in sets of 8. The occurrence of 5 or 6 is called a success. In what proportion of sets would you expect 3 successes? **(6)**

**OR**

**4.** a) Fit a parabolic curve to the following data. **(5)**

| | | | | | |
|---|---|---|---|---|---|
| x : | 2 | 4 | 6 | 8 | 10 |
| y : | 8.07 | 12.85 | 31.47 | 57.38 | 91.29 |

b) Calculate the coefficient of correlation between x and y using the following data: **(5)**

| x: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| y: | 9 | 8 | 10 | 12 | 11 | 13 | 14 | 16 | 15 |

c) Suppose that a manufactured product has two defect per unit of product inspected. Using prisson distribution, calculate the probabilities of finding a product without any defect, 3 defects, and 4 defects. (Given $e^{-2} = 0.1353$) **(6)**

## Unit - V

5. a) Find the least value of the integral. **(8)**

$$I = \int_P^Q \frac{\sqrt{1+y'^2}}{y}\,dx$$

Where P(-1,1) and Q(1,1) are points

b) Define Weak variations, strong variation, Extremal and Derive Euler's Equation and also other forms of Euler's Equation. **(8)**

**OR**

a) Show that the shortest distance between two points in a plane in a straight line. **(5)**

b) Find the shape of the curve of the given perimeter enclosing maximum area. **(5)**

c) Find the extremals of the functional and extremum value of $I = \int_0^2 (x-y')^2\,dx$

Subject to y(0) = 0 and y(2) = 4. **(6)**