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3E1704

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B. Tech. (Sem. III) (Main/Back) Examination, February - 2013
Ceramic Engineering
3CRE4 Mathematics : Paper - III (Common for EC, EIC, BM, AI, CR, PE)

Time : 3 Hours] [Total Marks : 80
[Min. Passing Marks : 24

*Attempt any **five** questions, selecting **one** question from each unit. All questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*
Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. NIL 2. NIL

UNIT - I

1 (a) If $f(t)$ is a periodic function with period $T>0$, then show that $L[f(t)]=\frac{1}{1-e^{-Ts}}\int_0^T e^{-st}f(t)dt$. 8

(b) Use laplace transform to solve $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$. Given that $u(0,t)=0=u(5,t)$, and $u(x,o)=10\sin 4\pi x$ 8

OR

1 (a) Compute laplace transform of the following functions :
(i) cos *at* cosh *at* 4
(ii) $e^{-2t}(5\cos 6t-4\sin 4t)$ 4

(b) Use laplace transform to solve :
$(D^2-3D+2)x=1-e^{2t}$, $x(0)=1, x^1(0)=0$ where $D\equiv\frac{d}{dt}$. 8

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UNIT - II

2 (a) Find the fourier series for $f(x)=x+x^2, -\pi<x<\pi$. Hence show that $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\ldots$

6+2

(b) Obtain the constant term and the coefficient of the first sine and cosine terms in the fourier series that represents y as given in the following table :

x:	0	1	2	3	4	5
y:	9	18	24	28	26	20

8

OR

2 (a) Show that :

(i) $Z(a^n)=\frac{z}{z-a}, n\geq 0$ 4

(ii) $Z(n^p)=-z\frac{d}{dz}\left[Z(n^{p-1})\right]$ where p is a positive integer and $n\geq 0$. 4

(b) Find z-transform of the following :

(i) $\cosh n\theta, n\geq 0$ 4

(ii) $\sinh n\theta, n\geq 0$ 4

UNIT - III

3 (a) Derive the relationship between fourier and laplace transforms. 8

(b) Find the fourier sine transform of $f(x)=e^{-x}, x\geq 0$. Also show that $\int_0^{\infty}\frac{x\sin mx}{x^2+1}dx=\frac{\pi}{2}e^{-m}$, where $m>0$.

8

OR

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3 (a) Express the function $f(x)=\begin{cases}1, |x|\le 1\\ 0, |x|>1\end{cases}$ as a fourier integral.

Hence evaluate $\int_0^\infty \frac{\sin\lambda}{\lambda}\cos\lambda x\, d\lambda$. Also deduce the value of

$\int_0^\infty \frac{\sin x}{x}dx$.

8

(b) Using fourier cosine tranform, solve :

$\frac{\partial\theta}{\partial t}=c^2\frac{\partial^2\theta}{\partial x^2}$ subject to the conditions

(i) $\theta=0$ when $t=0,\ x\ge 0$;

(ii) $\frac{\partial\theta}{\partial x}=-\mu$, a constant, when $x=0$ and $t>0$.

Assume that $\theta(x,t)$ and $\frac{\partial\theta}{\partial x}$ both tend to zero as $x\to\infty$.

8

UNIT - IV

4 (a) Show that the function $f(z)=u+iv$, where

$$f(z)=\begin{cases}\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, & z\ne 0\\ 0, & z=0\end{cases}$$

satisfy the cauchy - Riemann equations at the origin, yet $f'(0)$ does not exist.

8

(b) Determine the bilinear transformation which maps $z_1=0, z_2=1, z_3=\infty$ into $w_1=i, w_2=-1, w_3=-i$.

8

OR

4 (a) If $f(z)$ is analytic within and on a closed curve c and 'a' is any point inside c, then show that

$$f(a)=\frac{1}{2\pi i}\oint_c \frac{f(z)}{z-a}dz$$

where the integration along c is taken in anticlockwise direction.

8

(b) Use Cauchy's integral formula to evaluate the following integrals :

(i) $\oint_c \frac{\cos \pi z}{z-1} dz$ 4

(ii) $\oint_c \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$

where c is the circle $|z| = 3$. 4

UNIT - V

5 (a) Expand $\frac{1}{z\left(z^2 - 3z + 2\right)}$ in laurent series for the regions :

(i) $0 < |z| < 1$ 4

(ii) $1 < |z| < 2$ 4

(b) Find the residue of $\frac{z^2 - 2z}{(z+1)^2\left(z^2+4\right)}$ at all its poles in the finite plane. 8

OR

5 (a) Use Cauchy's residue theorem to evaluate $\oint_c \frac{z \cos z}{(z - \pi/2)^2} dz, c : |z-1| = 1$. 8

(b) Show that $\int_0^{2\pi} \frac{d\theta}{(5 - 3\sin\theta)^2} = \frac{5\pi}{32}$. 8