3E1704

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B.Tech. (Sem. III) (Main/Back) Examination, January - 2012
Electronics & Comm.
3EC1 Mathematics-III
(Common for Ceramic (3CRE4), 3EC1, 3AI1, 3EI1 & 3BM1)

Time : **3** Hours] [Total Marks : **80**

[Min. Passing Marks : **24**

Instructions to Candidates :

*Attempt any **five questions** selecting **one question** from each unit. All questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.*

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. Nil 2. Nil

UNIT - I

1 (a) Find the Laplace transform of $\sin\sqrt{t}$. Hence find the Laplace transform of $\dfrac{\cos\sqrt{t}}{\sqrt{t}}$.

(b) Solve : $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+y=t$ given that $y(0)=-3,\ y(1)=-1$

OR

1 (a) Find the inverse Laplace transform with the help of convolution theorem of $\dfrac{s}{\left(s^2+a^2\right)^2}$.

(b) Solve : $\dfrac{\partial u}{\partial t}=2\dfrac{\partial^2 u}{\partial x^2}$ where $u=u(x,t)$.

B.C. : $u(0,t)=0=u(5,t)$ and $u(x,0)=10\sin 4\pi x$

UNIT - II

2 (a) Find the Fourier series for the function defined as :

$$f(x) = \begin{cases} -1, & \text{for } -\pi \le x < 0 \\ 0, & \text{for } x = 0 \\ 1, & \text{for } 0 < x \le \pi \end{cases}$$

Hence prove that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$

(b) For z transform prove that

$$z(nu_n) = -z\frac{d}{dz}z(u_n)$$

with the help of this find the z transform of $n\,e^{-an}, n \ge 0$.

OR

2 (a) Obtain the constant term and the coefficients of first sine and cosine terms in the fourier expansion of y as given in the following table :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(b) Find the inverse z-transform of

$$f(z) = \frac{1}{(z-3)(z-2)}$$

If RDC is (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$.

UNIT - III

3 (a) Find the Fourier cosine transform of e^{-x^2}.

(b) Solve $\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}$ if (i) $V_x(0, t) = 0$ (ii) $V(x, 0) = \begin{cases} x, & 0 \le x < 1 \\ 0, & x > 1 \end{cases}$

and (iii) $V(x, t)$ is bounded $x > 0, t > 0$.

OR

3 (a) Find $f(x)$ if its Fourier cosine transform is $\frac{1}{1+s^2}$.

(b) Solve $\frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2}$, $x > 0, t > 0$

with B.C. : $\theta = \theta_0$ when $x = 0, t > 0$

and I.C. : $\theta = 0$ when $t = 0, x > 0$

UNIT - IV

4 (a) Define analytic function and derive Cauchy-Riemann conditions for analytic function and examine the nature of the function

$$f(z) = \frac{x^2 y^5 (x+iy)}{x^4 + y^{10}}, \ z \neq 0, f(0) = 0$$

in the region including the origin.

(b) If $(u-v) = (x-y)(x^2+4xy+y^2)$ and $f(z) = u+iv$ is an analytic function of $z = x+iy$ find $f(z)$ in terms of z.

OR

4 (a) Find the bilinear transformation which maps the points $z = 1, i, -1$ respectively on to the points $w = i, 0, -i$.

For this transformation, find the image of concentric circles $|z| = r, (r > 1)$.

(b) Verify Cauchy's theorem for the function $z^3 - iz^2 - 5z + 2i$ if C is the circle $(z-1) = 2$.

UNIT - V

5 (a) Obtain expansion for $\frac{z^2-4}{(z+1)(z+4)}$ which are valid, for the regions (a) $|z|<1$ (b) $1<|z|<4$ (c) $|z|>4$.

(b) Evaluate $\int_0^\infty \frac{1-\cos x}{x^2}\,dx$ by contour Integration.

OR

5 (a) Evaluate $\int_C \frac{z^2 e^{zt}}{z^2+1}\,dz$ where C is the circle $|z|=2$ and t is a quantity independent of z.

(b) Use method of contour integration to evaluate

$$\int_0^{2\pi} \frac{d\theta}{1+a^2-2\cos\theta},\ 0<a<1.$$
