

Time : 3 Hours]

[Maximum Marks : 80 [Min. Passing Marks : 24

Nil

Attempt any five questions, selecting one question from each unit. All questions carry equal marks. (Schematic diagrams must be shown wherever necessary.) Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1._____

UNIT - I

2.

1

1

(a) Find the coordinates of centre and radius of the circle

$$x^{2}+y^{2}+z^{2}-2y-4z=11$$
, $x+2y+2z=15$

(b) Find the equation of the right circular cone generated by straight line drawn from the origin to cut the circle through the three points (1,2,2) (2,1,-2) and (2,-2,1).

OR

- (a) Find the equation of the sphere which touches the plane 3x+2y-z+2=0 at the point (1, -2, 1) and also cuts the sphere $x^2+y^2+z^2-4x+6y+4 = 0$ orthogonally.
 - (b) Find the equation of a right circular cylinder whose generators are parallel to z axis and intersect the surface ax²+by²+cz²=1 and lx+my+nz=p.

2E1022]

1

[Contd...

8

8

8

8

UNIT - II

2

(a) Find out for what values of λ the equations

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$

and $x + 4y + 10z = \lambda^2$

have a solution and solve them completely in each case.

b) If
$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

Find the characteristic equation of A. Prove that A satisfies this equation and hence find A^{-1} .

OR

(a) Examine whether the following equations are consistent and solve them if they are consistent :

x+2y-z=3 3x-y+2z=1 2x-2y+3z=2x-y+z=-1

(b) Find the eigen values and the corresponding eigen vectors of the following matrix :

$$4 = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

UNIT - III

2

3

(a) Find by vector method the tangential and normal components of the velocity and acceleration.

(b) Show that the vector field defined by

 $\overrightarrow{A} = 2xyz^3 \widehat{i} + x^2z^3 \widehat{j} + 3x^2 yz^2 \widehat{k}$ is irrotational and find the scalar potential.

2E1022]

[Contd...

8

8

8

8

4

4

(c) Verify Gauss's Divergence theorem and show that

$$\int_{s} \overrightarrow{F} \cdot \hat{\mathbf{n}} \, \mathrm{ds} = \frac{1}{3} a^{5}, \text{ where } \overrightarrow{F} = (x^{3} - yz)\hat{\mathbf{i}} - 2x^{2}y\hat{\mathbf{j}} + 2\hat{k}$$

s is the surface of the cube bounded by the coordinate planes: x = y = z = 0; x = y = z = a

OR

- (a) Find the constants a and b so that the surface $ax^2-byz = (a+2)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point (1, -1, 2).
- (b) Find λ, μ and w if the following vector is irrotational $(2x + 3y + \lambda z)\hat{i} + (\mu x + 2y + 3z)\hat{j} + (2x + wy + 3z)\hat{k}$
- (c) Verify Stokes theorem for the function $F = x^2 \hat{i} + xy\hat{j}$ integrated round the square in the plane z = 0, whose sides are along the lines x = y = 0 and x = y = a.

UNIT - IV

- (a) The radial and transverse velocities of a particle are λr and μθ. Find its path and also its radial and transverse components of accelerations.
- (b) A particle of mass m is falling under gravity through a medium whose resistance is μ times the velocity. If the particle is released from the rest; show that the distance fallen through in time t is,

$$g\frac{m^2}{\mu^2}\left[e^{-\frac{\mu t}{m}}-1+\frac{\mu t}{m}\right]$$

OR

3

(a)

4

3

4

Prove that the angular acceleration in the direction of motion of a point moving in a plane is

$$\frac{\vartheta}{\rho}\frac{d\vartheta}{ds} - \frac{\vartheta^2}{\rho^2}\frac{d\rho}{ds}$$

2E1022]

[Contd...

8

4

4

8

8

8

8

(b) A particle of mass m is projected vertically under gravity, the resistance of the air being m k times the velocity; show that the highest height attained by the particle is,

$$\frac{V^2}{g} \Big[\lambda - \log(1+\lambda) \Big],$$

where V is the terminal velocity of the particle and λV is the initial vertical velocity.

UNIT - V

- 5 (a) Solve : (y + z)p + (z + x)q = x + y
 - (b) Solve : $z^2(p^2+q^2) = x^2+y^2$
 - (c) Solve in series

$$(2x + x^{3})\frac{d^{2}y}{dx^{2}} - \frac{dy}{dx} - 6xy = 0$$

OR

4

5 (a) Solve
$$: (x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$

(b) Solve :
$$(x^2 + y^2)(p^2 + q^2) = 1$$

(c) Apply Charpits method to solve

$$(p+q)(px+qy)-1=0$$

[70160]

8

4

4

8

4

4

8

2E1022]