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B. Tech. I-Sem. (Reback) Exam., Feb. 2013

Engineering Mathematics-I

(Common to all branches of Engg.)

Time: 3 Hours

Maximum Marks: 80

Min. Passing Marks: 24

Instructions to Candidates:

*Attempt any **five questions,** selecting **one question** from **each unit.** All questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.*

Use of following supporting material is permitted during examination.

1. Nil
2. Nil

UNIT – I

Q.1. (a) Show that the asymptotes of the curve.
$x^3 - 2y^3 + 2x^2y - xy^2 + xy - y^2 + 1 = 0$. Cut the curve in three points which lie on the line x-y+1=0 [8]

(b) Trace the curves:

(i) $y(x^2 + 4a^2) = 8a^3$, where $a > 0$ [4]

(ii) $r = a + b\cos\theta$, where $a < b$ [4]

OR

Q.1. (a) Show that the points of inflexion of the curve
$y^2 = (x-a)^2(x-b)$ lie on the straight line $3x + a = 4b$. [8]

(b) For the curve.
$y = a\log\sec\left(\dfrac{x}{a}\right)$, prove that the chord of curvature parallel to the axis y is of constant length. [8]

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UNIT – II

Q.2. (a) State and prove Euler's theorem on homogeneous functions and verify it for th function $f(x,y) = ax^2 + 2bxy + by^2$. [5

(b) The period of a simple pendulum is given by $T = 2\pi\sqrt{\left(\frac{\ell}{g}\right)}$.

Find the maximum approximate error in T due to possible errors upto 1 perc in ℓ and 2.5 percent in g. [5

(c) In a plane triangle ABC, find the maximum value of $\cos A \cos B \cos C$ [6

OR

Q.2. (a) if $k = \log(x^3 + y^3 + z^3 - 3xyz)$

Find the value of

$$\frac{\partial^2 k}{\partial x^2} + \frac{\partial^2 k}{\partial y^2} + \frac{\partial^2 k}{\partial z^2}$$ [5]

(b) The diameter and altitude of a right circular cylinder are measured as 4cm. and 6cm. respectively. If the possible error in each measurement is 0.1cm., find maximum possible errors in the values of volume and the lateral surface. [5]

(c) Describe the Lagrange's method of undetermined multipliers to find the maximum or minimum values of a function. [6]

UNIT – III

Q.3. (a) The lemniscates

$r^2 = a^2 \cos 2\theta$ revolves about a tangent at the pole. Find the whole surface of the solid generated. [5]

(b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 3$ and z=0. [5]

(c) Evaluate

$$\int_0^{\frac{\pi}{2}} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta$$ [3]

Hence prove that

$$\sqrt{\pi}\,\Gamma(2m) = 2^{2m-1}\Gamma(m)\Gamma(m+\frac{1}{2})$$ [3]

OR

Q.3. (a) Find the volume of the solid generated by the revolution of an arch of the cycloid.

$x = a(\theta - \sin\theta)$

$y = a(1 - \cos\theta)$

about the axis of x [5]

(b) Evaluate $\int_0^1 \left[\int_x^{\sqrt{(2-x^2)}} \frac{x}{\sqrt{(x^2+y^2)}} dy \right] dx$

by changing the order of integration [5]

(c) Evaluate the integral

$$\int_0^1 x^m (1-x^n)^p \, dx$$ [4]

in terms of Beta function and hence obtain the value of the integrai

$$\int_0^1 x^5 (1-x^3)^{10} \, dx$$ [2]

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UNIT – IV

Q.4. Solve the following differential equations.

(a) $\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$ [5]

(b) $(x - y - 2)dx - (2x - 2y - 3)dy = 0$ [5]

(c) $(D^4 + 2D^2 + 1)y = x^2 \cos x$ [6]

OR

Q.4. Solve the following differential equations.

(a) $y \sin 2x dx - (1 + y^2 + \cos^2 x)dy = 0$ [5]

(b) $\frac{xdx + ydy}{xdy - ydx} = \sqrt{\left(\frac{a^2 - x^2 - y^2}{x^2 + y^2}\right)}$ [5]

(c) $(D^3 - D^2 + 4D - 4)y = 3\cos 2x + 4\sin 3x$ [6]

UNIT – V

Q.5. Solve the following differential equations:

(a) $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$ [8]

(b) $\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2y \cos^3 x = 2\cos^5 x$ [8]

OR

Q.5. (a) Solve:

$$\frac{d^2y}{dx^2} + 2x \frac{dy}{dx} + (x^2 + 1)y = x^3 + 3x$$ [8]

(b) Solve by the method of variation of parameters:

$$\frac{d^2y}{dx^2} + a^2 y = \sec ax$$ [8]
